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AN INVESTIGATION INTO THE BEAM FROM A STANDARD LIGHTHOUSE LENS

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ABSTRACT. The distribution of light from a third order Fresnel panel was measured, using two different light sources, a standard petroleum vapour burner and a 3 kW. lighthouse cruciform lamp.

A complete theoretical discussion of the factors affecting beam candle-powers is worked out and tables are provided for the calculation of the beam candle-power from any apparatus using any light source.

INTRODUCTION

THIS investigation was carried out at the suggestion of Mr J. P. Bowen, Engineer-in-Chief to the Corporation of Trinity House. The problem set by Mr Bowen can be conveniently divided into two main parts:

1. What is the beam power given by the half panel of the third order apparatus designed for Hartland Point, using
   (*a*) a standard petroleum vapour burner,
   (*b*) a 3 kW. lighthouse cruciform lamp?
2. In computing the power of a lens it is usual to employ the formula

   Beam power = constant × area of lens × brightness of source.

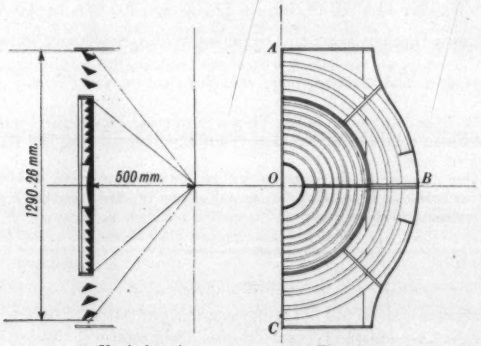
Is this "constant" truly constant or, if not, what are appropriate values of it which should be used for various illuminants and different parts of the lens?

The lens for Hartland Point consists of various panels of different angles, of which one is exactly half the standard third order lens, together with certain reflectors. These reflectors offer a useful basis for investigation into the beam from such prisms. Fig. 1 shows the panel in question.

SECTION I

(*a*) The Beam from the Lens using a "Chance" 55 mm. Autoform Burner

The lens was erected at one end of the lighthouse shops on a rotating table with a "Chance" 55 mm. burner supplied with the usual valves and gauges. A photometer bench, using a standard 10-candle pentane lamp and a flicker photometer loaned by Trinity House, was erected at the opposite end of the shop, the distance from the lens to the pentane lamp being 240 feet approximately. This is well beyond the "cross-over" point for this size of illuminant. At right angles to the line through the photometer a second photometer bench with a second pentane lamp and a Lummer-Brodhun photometer was arranged. This photometer measured the candle-power of the petroleum vapour burner itself, so that by using two observers a simultaneous reading of the lens and light source could be taken.



Vertical section Elevation

Fig. 1. Panel of lens in Hartland Point apparatus

Preliminary measurements made in two or three horizontal directions showed that the candle-power of the mantle was practically uniform and so that the results from the lens should correspond to the polar diagram (see Section III (*a*)) the mantle was run at the same brightness as that used for the polar curves. When the control reading showed a variation from this value, a correction was applied to the beam value in the ratio of the standard candle-power to the observed candle-power.

Measurements were made at a number of positions on each side of the optical axis, this latter being determined by a theodolite setting, the lens centre and light centre being placed in line with the photometer bench. A scale read by a telescope fixed on the rotating table enabled the angular position of each reading to be determined. The half-lens was on the right-hand side when looking along the

beam from the apparatus. The readings obtained when the bare mantle gave a reading of 1140 candles are plotted in Fig. 2.

The brightness of the mantle measured horizontally was 39·1 candles per sq. cm. and the total area of the lens was 5160 sq. cm., so that assuming the beam power to be 125,000 the factor for the whole lens is

$$\frac{125{,}000}{39{\cdot}1 \times 5160} = {\cdot}625 \text{ on this basis.}$$

This is discussed in detail later.

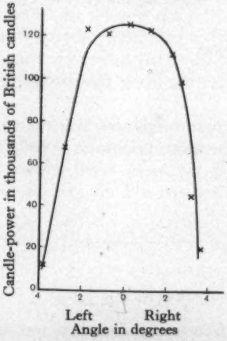


Fig. 2. Distribution of candle-power in the beam, using a "Chance" 55 mm. auto-form petroleum vapour burner

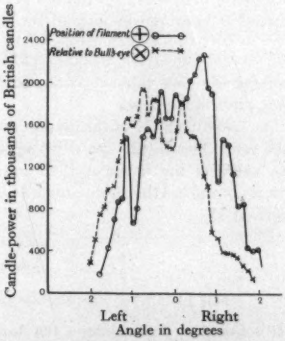


Fig. 3. Distribution of candle-power in the beam, using a 3 kW. cruciform electric lamp

(*b*) The Beam from the Lens, using a 3 kW. Cruciform Electric Lamp

For the comparison lamp it was decided to use a 500-watt tubular projector lamp, as this gives a much higher candle-power (about 1000 candles) than the pentane standard, eliminates any colour differences, and, as experiments detailed in Section II showed, varies in candle-power with change of voltage in the same manner as the 3 kW. lamp. Thus there was no necessity to correct the readings for voltage changes although a voltmeter was included in the circuit.

The 3 kW. lamp was centred in the apparatus on the same table as mentioned in Section I (*a*) and the standard comparison lamp was erected at the maximum distance available—some 380 feet away. A Lummer-Brodhun photometer was used for the measurements. The initial setting to find the optical axis and the arrangements for angular measurements of the lens position were as before. The supply to both lamps was taken from a transformer working at 80 volts and 25 cycles. This transformer had five tappings between 77–83 volts for calibration purposes. The voltage during the readings, however, only varied over a range of ·7 volt. Photometric readings were taken at intervals of approximately 6 minutes of angle and

two series were made, one with the lamp filament in the × position and the other in the + position. The results are plotted in Fig. 3, assuming the value for the comparison lamp given by the General Electric Co., viz. 1060 candles at 80 volts.

The beam from this lamp is obviously a beam from a number of separate light sources, each filament giving its own beam aided to some extent by the spread from its neighbours. Thus the maxima should be given by the brightness of single filaments. In the + position 10 filaments of 2 mm. diameter and 20 mm. length project their separate beams to form the source. The candle-power is 4950 candles or the brightness, say, 1250 candles per sq. cm. The whole area is 5160 sq. cm. and the possible beam power without correction is

$$5160 \times 1250 = \mathbf{6{,}450{,}000}.$$

Assuming the peak value at 2,200,000, the factor for the whole lens is **·340**. This only applies to the peaks.

The usual method of calculation is to take a mean brightness over the whole of the grid. This is $38 \times 20 = 7{\cdot}6$ sq. cm., and the mean brightness = **650**. The mean value of the factor is then calculated from the mean beam power—say 1,700,000—and has the approximate value **·510**. This matter is thoroughly discussed in Section IV.

SECTION II

The Effective Brightness of Different Parts of the Lens

It is necessary to consider a few fundamental facts concerning these measurements before describing the experimental lay-out. The details required from the experiments are obviously the contributions to the total beam given by a small area in different portions of the lens. Theoretically, the photometer should remain in the same position, that is, at as large a distance from the lens as when the whole beam was measured. This is physically impossible, owing to the very low illumination given by such a small element. It thus becomes necessary to bring the photometer relatively close to the lens and at such a short distance the divergence of the light is insufficient to make the beam cross the optical axis of the apparatus. It is therefore essential to bring the photometer on to the imaginary line joining the element being measured and the previous position of the photometer. This is essentially bringing the photometer on to a line parallel to the optical axis of the apparatus through the small element being measured. In the experiments carried out this result was obtained by using two mirrors placed at 45° to the axis of the lens, one being fixed on the axis and the other opposite the aperture in the screen. This obviated the necessity of moving the photometer except for the purpose of obtaining a balance.

A further difficulty lies in the fact that each element of the lens projects a beam essentially of the same shape as the major beam. In other words, there is a small area of practically uniform power at the centre with a rapid falling away of intensity all round. In an optically perfect apparatus using a uniform spherical source, this

brightest area is projected parallel to the axis of the lens and is the contribution of the element to the whole beam. Slight errors in the setting of the mirrors, either in angle or in absolute position relative to the aperture in front of the lens, displace this area and so traverses were made across each little beam in two directions at right angles in order to find the maximum reading.

The statement above that each element projects a beam similar to the major beam suggests another difficulty when using commercial light sources. With an electric lamp, for instance, separate images of each filament are projected and give lines of light across the photometer field, fringed with colour in the case of the refractors. In the case of a petroleum vapour burner mantle the top prisms project a black spot corresponding to the hole in the top of the mantle. In both cases the variation of intensity over the small beam is so great as to render accurate measurement impossible.

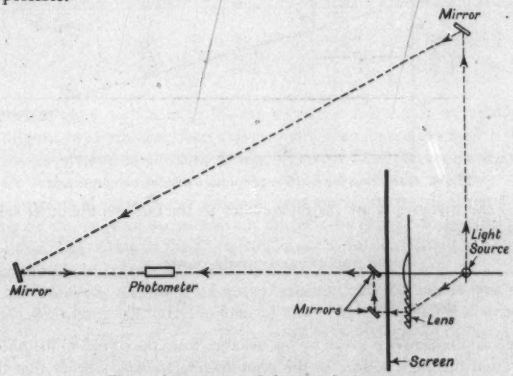


Fig. 4. Arrangement of photometric apparatus

For these reasons it was found necessary to use a special light source, an opal electric bulb, for this exploration over the lens. Only relative contributions to the total beam were required, so that the power of each portion of the lens relative to the power of the same area of naked lamp gave the desired information.

Fig. 4 shows the lay-out of the apparatus. One side of the photometer was illuminated by a beam from a small aperture in a screen covering the whole of the lens, reflected by the two mirrors so that its ultimate path was axial to the lens. The other side of the photometer was illuminated by light from the naked bulb reflected from mirrors as indicated. This arrangement had the advantages that there was no colour difference in the photometer and that variations in voltage or power of the source did not affect the point of balance.

The readings are obtained from the distances from the screen to the photometer on the one side and the lamp to the photometer on the other. This is based

on the following considerations. Let AD (Fig. 5) represent the lens face, B the aperture in the screen, and C the photometer. Then the photometer is illuminated by the light falling on it from the area corresponding to AD while the aperture only measures ED. Thus the light is too great in the proportion of AD^2/ED^2. Now, if l_1 = the distance from lens to photometer and l_2 = the distance from lens to screen,

$$\frac{AD}{ED} = \frac{l_1}{l_1 - l_2}.$$

Thus the true power given by the area of the aperture

$$= \text{power measured} \times \left(\frac{l_1 - l_2}{l_1}\right)^2.$$

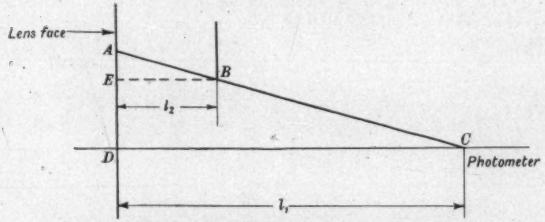


Fig. 5. Correction for position of aperture relative to light source

If l_3 = the distance from the photometer to the bulb on the other side of the photometer, then

$$\text{the uncorrected candle-power} = \frac{l_1^2}{l_3^2},$$

or

$$\text{the corrected power} = \left(\frac{l_1 - l_2}{l_1}\right)^2 \cdot \left(\frac{l_1}{l_3}\right)^2 = \left(\frac{l_1 - l_2}{l_3}\right)^2,$$

or the true candle-power is given by the distance from the screen to the photometer and not from the light source to the photometer, *provided always* that the light source subtends a larger angle at the photometer than the aperture does.

The half angle of the bulb at the centre of the lens was ·087 radian (= 5°). Four readings were taken using the naked opal bulb; these are given in Table I.

TABLE I. *Values of the ratio* $\left(\frac{l_1 - l_2}{l_3}\right)^2$

l_1 ins.	l_2 ins.	l_3 ins.	$\left(\frac{l_1 - l_2}{l_3}\right)^2$
210·5	10·0	636·5	·0992
215·5	16·0	631·0	·0994
220·5	21·5	626·5	·1008
209·0	32·0	636·5	·0772

The mean value ·1000 was taken as the value for the naked light source, since the last reading was obviously in error due to the distance l_2 being too great.

The values obtained for the different refractors and reflectors, relative to 1·00 for the naked bulb, are given in Table II.

TABLE II. *Brightness of prism faces relative to brightness of source*

Refractor	Vertically above *OA* (Fig. 1)	Vertically below *OC*	Horizontal *OB*	Mean from graph
1	—	—	·826	·826
2	·808	·802	·808	·815
3	·778	·802	·773	·800
4	·778	·791	·778	·787
5	·778	·784	·773	·772
6	·758	·737	·754	·754
7	·723	·728	·745	·725
8	·683	·681	·684	·683
9	·618	·606	·620	·615
Reflector				
1	·707	·683	·698	·696
2	·703	·687	·687	·693
3	·698	·685	—	·691

The values given in Table II for the refractors are the mean values for the relative brightness of each lens face, but they still need correction to give the factor desired for calculations of the total power. The area used for computing is in the area of the *inside* face of the lens, i.e. the whole area of glass available. A beam, however, consists of a number of dark and light zones, the width of the dark zones increasing as the angular distance from the axis increases. Reference to Fig. 1 indicates this. The values in the above table need to be diminished in order to give factors applicable to the whole area of light and dark zones.

Let *ABC* (Fig. 6) represent a Fresnel refractor, *AC* the inside face used as a basis for calculation, and *AD* the area of the light zone which is measured in Table II. To give a *mean* value, the factor needs to be reduced in the ratio *AD*/*AC*. This may be calculated as follows: Let $\angle BAC = \psi$ and $\angle BCX = r$. Then it may be shown that

$$\frac{AD}{AC} = \frac{1}{1 + \tan\psi \tan r},$$

and

$$\tan\psi = \frac{\mu \sin r}{\mu \cos r - 1} \quad \text{and} \quad \sin\theta = \mu \sin r,$$

where μ = refractive index.

Hence

$$\frac{AD}{AC} = \frac{\cos r\,(\mu \cos r - 1)}{\mu - \cos r}.$$

Values of this ratio are given in Table III for various values of θ.

The product of the appropriate value of *AD*/*AC* and the mean value for the refractors in Table II gives finally the factor for an element in each zone for the case of an opal bulb subtending a half angle of ·087 radian (= 5°). The values of this factor are given in Table IV and are plotted in Fig. 7.

TABLE III. *Values of the ratio AD/DC*

θ	AD/AC
°	
0	1·00
5	·98
10	·95
15	·91
20	·85
25	·79
30	·71
35	·63
40	·55
45	·48

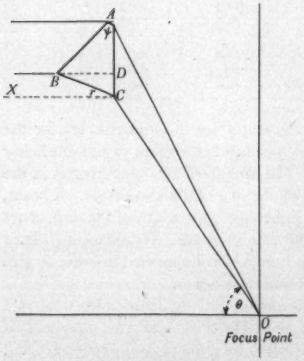


Fig. 6. Correction of measurements for dark zones in refractors

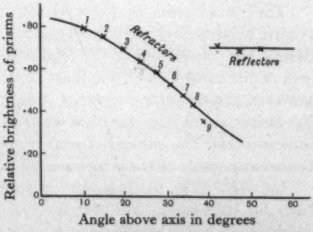


Fig. 7. Measured and calculated brightness of refractors and reflectors (experimental results shown by crosses)

TABLE IV. *Brightness of lens relative to source: Refractors*

Experimental values of the factor for the case of a semi-angle of ·087 radian

θ	Factor (i.e. relative brightness)
° ′	
10 0	·787
14 50	·750
19 45	·690
23 45	·635
27 15	·580
30 30	·527
33 30	·474
36 0	·427
38 45	·357

The mean value of the brightness of the lens is not the arithmetical mean of the above, as the outer zones cover more area than the inner. The area is in fact proportional to tan θ (see Section IV), and using these factors the mean brightness of the whole of the refractors is ·526.

In computing the area of the reflectors of this particular lens only the external glass face has been measured as the arrangement is somewhat abnormal. The figures therefore need no correction and give a mean value of ·694 for all the reflecting prisms (see Fig. 7). This value only applies to the particular opal bulb used in the experiments, but further reference to this is made later.

SECTION III

(*a*) The Polar Curves of the Illuminants

The polar curves of the petroleum vapour burner mantle and the opal gas-filled lamp were determined by means of an arrangement of two mirrors—the standard method described in textbooks on photometry. A compensating colour screen was necessary in the case of the petroleum vapour burner mantle, when using a pentane standard.

The curves for the 3 kW. lamp were supplied by the General Electric Co. for a lamp of the same type. They do not, however, refer to the actual lamp used, but have been adjusted to give the same horizontal candle-power.

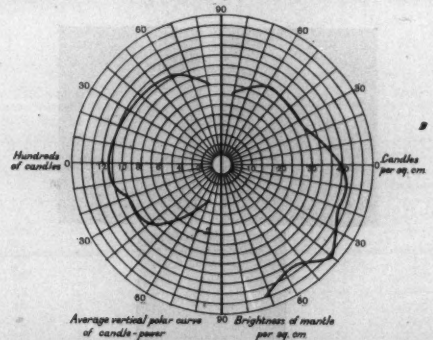


Fig. 8. Polar curves of "Chance" 55 mm. autoform petroleum vapour burner

Petroleum Vapour Burner

The area of the petroleum vapour burner mantle was also measured in a number of positions by photographing the image given in the mirrors. The negatives were enlarged to full size and the prints measured with a planimeter. The height of the image was also taken. From these data the values of the brightness given in Table V were obtained. The results are plotted in Fig. 8.

TABLE V. *"Chance" 55 mm. autoform petroleum vapour burner mantle*

Angular position	Candle-power	Brightness Candles per sq. cm.	Height of image mm.
°			
90 above	—	—	—
80	810	24·2	66
70	925	26·9	66
60	1025	30·2	68
50	1040	29·6	70
40	1050	29·5	68
30	1060	29·6	66
20	1080	31·6	62
10	1120	35·4	56
0 horizontal	1140	39·1	54
− 10 below	1100	41·7	50
− 20	1000	41·2	44
− 30	990	43·8	39
− 40	940	47·7	34
− 50	750	42·8	29
− 60	580	41·7	24
− 70	400	46·0	20
− 80	—	—	—
− 90	—	—	—

The horizontal readings were found to be sensibly constant.

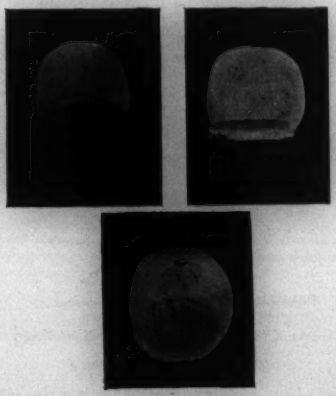


Fig. 9. Photographs of "Chance" 55 mm autoform petroleum vapour burner mantle

It should be noticed that the brightness decreases regularly from the bottom of the mantle to the top and also as the visible area decreases the brightness increases. Two factors contribute to this result. Since the mantle is woven the material is

partially transparent, but as the line of sight becomes nearer to grazing incidence this transparency disappears. The flame is hotter near the top of the burner and the bottom of the mantle is necessarily at a higher temperature than the top. Three typical photographs of the mantle are shown in Fig. 9.

3 *kW. Cruciform Electric Lamp*

The results of the measurements with the 3 kW. cruciform lamp are given in Table VI and are plotted in Fig. 10.

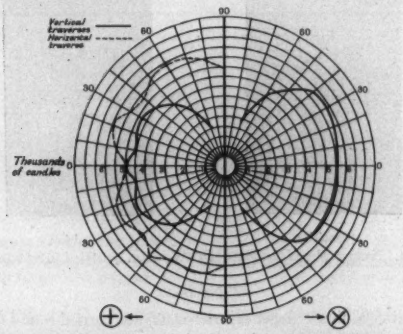
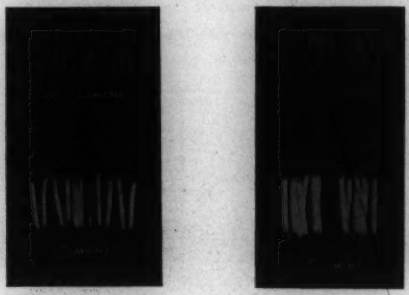


Fig. 10. Polar curves of 3 kW. 80 volt cruciform filament lamp

TABLE VI. 3 *kW. cruciform lamp*

Angle	Mean horizontal traverse	Vertical traverse	
		+ position	× position
°	Candle-power	Candle-power	Candle-power
90	4950	—	—
80	5360	—	—
70	5780	2140	2350
60	5870	3210	4120
50	5760	4040	5000
40	5320	4500	5750
30	5490	4820	5820
20	5940	4700	5750
10	5600	4550	5640
0	4950	4950	5575
− 10	5320	4550	5640
− 20	5830	4810	5640
− 30	5770	4820	5630
− 40	5370	4420	5520
− 50	5880	3800	4870
− 60	5480	2810	3400
− 70	5600	2140	2250
− 80	5480	—	—
− 90	4950	—	—

Photographs of the filament grid in the two positions are shown in Fig. 11. The "ghost" image was found by measurement to contribute about 5 per cent. to the total candle-power, but the data above have been corrected to read the values without the "ghost," as this image cannot contribute to the beam given by the lens.



+ position × position

Fig. 11. Photographs of filaments of 3 kW. G.E.C. gas-filled lighthouse lamp

Opal Bulb

The opal bulb (Phillips' "Argenta" 100-watt) was turned round for the readings so that the same position of the surface faced the section of the lens being measured. Polar readings over this section showed the candle-power to vary only between 89·5 and 91·0. This was therefore assumed to have a constant brightness over the range being used.

(*b*) The Effect of Oil Pressure on the Candle-power of a Petroleum Vapour Burner Mantle

In order that the candle-power of the mantle might be maintained at a given value, it was necessary to investigate the change of candle-power (and hence brightness) with change of pressure. A second gauge was inserted in the pipe leading into the vaporiser chamber, since the pressure here is controlled by the micrometer valve and is independent of the container pressure, provided the latter is high enough. A series of readings at different pressures was taken and the results are plotted in Fig. 12. It is interesting to note the very definite maximum in candle-power at a pressure of 40 lbs. per sq. inch.

(*c*) The Variation in Candle-power with Voltage

Graphs were supplied by the General Electric Co. with each lamp showing change of candle-power with change of voltage. The error introduced over a voltage range of 75 to 85 by assuming that the relative candle-power remained constant is given in Table VII from the General Electric Co. data. The corresponding

figures found experimentally from some sixty readings at different voltages are given in the last two columns of Table VII.

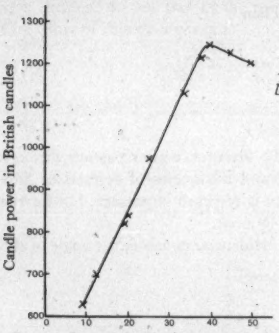


Fig. 12. Variation of candle-power of petroleum vapour burner with pressure

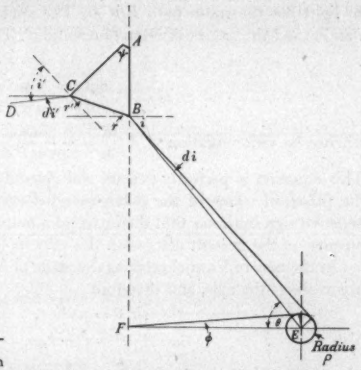


Fig. 13. Diagram of pencil of rays refracted through a prism

TABLE VII. *Variation in candle-power with voltage*

Volts	Comparison lamp	Error in 3 kW. lamp	Error found by experiment	
			+ position	× position
	candle-power	per cent.	per cent.	per cent.
75	848	− 2·0	− 1·7	− 1·7
77·5	940	− 1·1	+ ·4	− ·7
80	1060	+ 1·6	− 1·3	+ ·3
82·5	1160	+ ·5	+ 1·5	+ 1·3
85	1300	+ 1·6	+ 1·0	+ ·8

As the total voltage range was only ·7 volt during the readings no correction was applied.

SECTION IV

A THEORETICAL DEDUCTION OF THE BRIGHTNESS OF THE LENS FACE

Various papers have been published bearing on this problem; some of them assume a parallelism between reflecting and refracting apparatus which cannot be maintained and none of them, so far as the author is aware, consider the effect on the candle-power of the chromatic aberration which is inevitably present in refracting systems.

(*a*) Refracting Systems

Referring to Fig. 13, the refracting angle of the prism is ψ and a small pencil of light has the mean path *EBCD*. The angles of incidence and emergence are i and i' and the angles of refraction r and r'. Then

$$\psi = r + r' \qquad \ldots.(1),$$

$$\left.\begin{aligned} \sin i &= \mu \sin r \\ \sin i' &= \mu \sin r' \end{aligned}\right\} \qquad \ldots.(2),$$

whence by differentiation

$$\frac{di'}{di} = -\frac{\cos i \cos r'}{\cos i' \cos r} \qquad \ldots.(3).$$

This equation is perfectly general and gives the divergence after passage through the prism in terms of the divergence before and the angles of refraction. The negative sign indicates that the image of a point is reversed in passage, but for the purpose of the present discussion this may be ignored.

In the case of Fresnel sections the angle of incidence i equals θ, the angle of the prism above the axis, and therefore

$$\sin \theta = \mu \sin r.$$

Also

$$r' = i' - r,$$

whence

$$\frac{di'}{di} = \frac{\cos \theta \cos (i' - r)}{\cos i' \cos r} \qquad \ldots.(4).$$

This may be further reduced by substitution from equations (1) and (2) to the form

$$\frac{di'}{di} = -\cos \theta \left[\frac{\mu^2 - \sqrt{(\mu^2 - \sin^2 \theta)}}{\mu^2 - \sin^2 \theta - \sqrt{(\mu^2 - \sin^2 \theta)}}\right] \qquad \ldots.(5)$$

which is given by Walsh*.

Ribière† gives the empirical equation

$$\frac{di'}{di} = -\frac{1}{\cos \theta} \qquad \ldots.(6),$$

which agrees well with the numerical values deduced from equation (5). The values of di'/di derived from equations (5) and (6) are given in Table VIII.

Table VIII. *Values of di'/di for Fresnel prisms:* $\mu = 1{\cdot}5130$

θ	di'/di	
	From equation (5)	From equation (6)
°		
0	1·00	1·00
5	1·00	1·00
10	1·02	1·01
15	1·05	1·03
20	1·09	1·06
25	1·15	1·10
30	1·22	1·15
35	1·30	1·22
40	1·38	1·30
45	1·49	1·41

* J. W. T. Walsh, *Proc. Opt. Conv.* 1926 (385). † Ribière, *Phares et Signaux Maritimes*, p. 65.

(*b*) Divergence due to Dispersion of the Glass

In the preceding section the differentiation was carried out with μ constant, but actually μ varies for different wave-lengths. It follows that a pencil of white light incident on one face of the prism emerges as a coloured pencil on the other. The value of this divergence is

$$\frac{di'}{d\mu} = \frac{\mu \sin\theta}{\mu^2 - \sin^2\theta - \sqrt{(\mu^2 - \sin^2\theta)}} \qquad \ldots.(7).$$

Numerically the following expression gives a close approximation to equation (7):

$$\frac{di'}{d\mu} = \frac{2\tan\theta}{\cos\theta} \qquad \ldots.(8).$$

The values of $di'/d\mu$ derived from equations (7) and (8) are given in Table IX.

Table IX. *Values of* $di'/d\mu$ *for* $\mu = 1{\cdot}5130$

θ	$di'/d\mu$	
	From equation (7)	From equation (8)
°		
0	·00	·00
5	·17	·17
10	·35	·35
15	·52	·55
20	·73	·77
25	·96	1·03
30	1·22	1·33
35	1·54	1·71
40	1·92	2·19
45	2·26	2·83

The total external divergence di' of a pencil of initial divergence di is thus given as a sum of equations (5) and (7), since it is immaterial that the image is reversed. For lighthouse crown $d\mu$ may be taken as half the total dispersion between 4000 and 7000 A.U., i.e. ·010.

(*c*) The Intensity of the Beam from Fresnel Revolving Apparatus

The following discussion is based on Ribière's* treatment with certain additions and modifications. For the preliminary solution a uniform spherical source (realised, experimentally, by an opal bulb) is assumed. Let the radius be ρ and the focal distance ϕ. Then from Fig. 13

$$di = \frac{\rho\cos\theta}{\phi} \qquad \ldots.(9).$$

Then, if

di_R = angle subtended by source at angle θ considered radially

and di_T = angle subtended by source at angle θ considered tangentially

$$di'_R = \frac{di}{\cos\theta} + 2d\mu\frac{\tan\theta}{\cos\theta} \text{ from equations (6) and (8)}$$

and $di'_T = di_T$ since incidence is normal.

* *Ibid.* p. 65.

The emergent beam from a small element of refractor is therefore an elliptic cone measured by $\pi di''_T di''_R$.

Consider an elementary zone between θ and $\theta + d\theta$. The light falling on this zone is

$$\pi\rho^2 B \,.\, 2\pi \sin\theta \, d\theta$$

where B = brightness of the source. If K represents the transmission factor of the glass (including reflection and absorption losses) the intensity of the emergent cone is given by

$$\frac{2\pi^2\rho^2 KB \sin\theta d\theta}{\pi di''_T di''_R} = \frac{2\pi\rho^2 KB \sin\theta d\theta}{di_T di''_R} \qquad \ldots.(10).$$

But from equation (9) $\qquad di_T = \frac{\rho}{\phi}\cos\theta = di_R$

for a spherical source; thus equation (10) reduces to

$$2\pi\phi^2 KB \frac{\tan\theta}{\cos\theta}\frac{di_R}{di''_R}\, d\theta \qquad \ldots.(11).$$

The area of the inside face of the lens which contributes this beam is

$$\frac{d}{d\theta}\pi\phi^2\tan^2\theta = 2\pi\phi^2\frac{\tan\theta}{\cos^2\theta}\, d\theta \qquad \ldots.(12),$$

and the brightness of the lens = (11)/(12)

$$= KB\cos\theta\frac{di_R}{di''_R} \qquad \ldots.(13).$$

The total power of the whole beam is obviously from (11)

$$2\pi\phi^2 B\int_0^{\theta_m} K\frac{\tan\theta}{\cos\theta}\frac{di_R}{di''_R}\, d\theta \qquad \ldots.(14),$$

where θ_m is the maximum angle of the outer refractor, but the expression is too complex to lend itself to any but graphical methods of integration.

In order to find the required factor for computing the beam power from the area of the lens, the mean value of the brightness is needed. This is not the arithmetical mean of the values computed from equation (13), as there is a zone for each value of θ, which obviously increases as θ increases.

By making use of the identity

$$\pi\phi^2\tan^2\theta_m = 2\pi\phi^2\int_0^{\theta_m}\frac{\tan\theta}{\cos^2\theta}\, d\theta$$

equation (14) may be written

$$\frac{\pi\phi^2\tan^2\theta_m \times \int_0^{\theta_m} 2\pi\phi^2 BK\frac{\tan\theta}{\cos^2\theta}\cos\theta\frac{di_R}{di''_R}\, d\theta}{2\pi\phi^2\int_0^{\theta_m}\frac{\tan\theta}{\cos^2\theta}\, d\theta}$$

$$= \frac{\pi\phi^2\tan^2\theta_m\int_0^{\theta_m}\frac{\tan\theta}{\cos^2\theta}KB\cos\theta\frac{di_R}{di''_R}\, d\theta}{\int_0^{\theta_m}\frac{\tan\theta}{\cos^2\theta}\, d\theta}$$

= area of lens face × mean brightness of lens.

Thus the value of the factor required for computation is

$$\frac{\int_0^{\theta_m} \frac{\tan\theta}{\cos\theta} K \frac{di_R}{d\theta} d\theta}{\int_0^{\theta_m} \frac{\tan\theta}{\cos^2\theta} d\theta} \qquad \ldots\ldots(15).$$

(*d*) The Value of *K* (Transmission Factor of the Glass)

The light lost in passage through a prism depends on the angles of incidence and refraction and hence on the refractive index and the amount absorbed. For the former, Fresnel's formula has been used, viz.

$$\frac{I}{I_0} = \frac{1}{2}\left[\frac{\sin^2(i-r)}{\sin^2(i+r)} + \frac{\tan^2(i-r)}{\tan^2(i+r)}\right],$$

where I = light reflected from one surface and I_0 = light incident on the surface, and the loss by absorption has been taken to be 4 per cent. per cm. As the mean path in the refractors is 2 cm., this means an absorption of 8 per cent. The values of K for different values of θ are given in Table X.

Table X. *Values of K for refractors (third order section)*

θ	K
°	
0	·84
5	·84
10	·84
15	·84
20	·84
25	·83
30	·82
35	·80
40	·78
45	·74

These values of K were used in calculating the brightness of the refractors, obtained from equation (13), for various values of θ, and in calculating the mean brightness of the refractors, obtained from equation (15), between θ_m and θ_0. The results of these calculations are plotted in Figs. 14 and 15.

Fig. 7 gives the results of the calculation together with the measured brightness for different zones of the lens. The agreement is very good, so that the theoretical deduction may be taken as being based on sound premises.

(*e*) The Beam from the Reflectors

Since the light is totally reflected and suffers two refractions in opposite senses, it appears that there is no chromatic aberration. Also an incident beam of angle di emerges with its angle unchanged. Consider an element of a reflector $d\theta$ at an angle θ above the horizontal focal plane. It is assumed that the prism is vertically over the refractors as in the case of the Hartland Point apparatus. The calculation

follows the lines indicated in the case of the refractors and leads to the same result, except that now

$$di'_R = di'_T = di = \frac{\rho}{\phi}\cos\theta,$$

so that the emergent cone is πdi^2, and

$$\text{the intensity} = 2\pi\phi^2 KB\frac{\tan\theta}{\cos\theta}\,d\theta \qquad \ldots.(16).$$

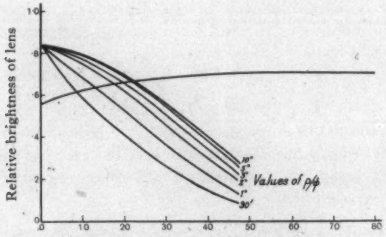


Fig. 14. Relative brightness of lens for various values of ρ/ϕ for a uniform spherical source

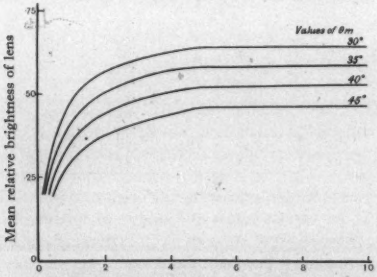


Fig. 15. Mean relative brightness of lens between θ_m and 0°, i.e. value of factor for uniform spherical source

The area of the lens producing this cone of light is

$$2\pi\phi^2\frac{\tan\theta}{\cos\theta}\,d\theta,$$

since the inside face and outside face are equal. Hence

$$\text{the brightness} = KB \qquad \ldots.(17)$$

and is independent of the angle. It should be noted that this derivation covers only

the actual glass, and in computing the area allowance must be made for the dark zones.

The value of K depends on two reflections and on a light path of 5·5 cm. The absorption of this length of glass is $\cdot 04 \times 5 \cdot 5 = \cdot 22$, and Table XI gives the angles of reflection for different values of θ. The final value of K is calculated as before from these data.

TABLE XI. *Values of K for reflecting prisms*

θ	i (approx.)	K
°	°	
0	63	·56
10	57	·62
20	51	·66
30	44	·68
40	36	·69
50	29	·70
60	21	·70
70	14	·70
80	7	·70
90	0	·70

These values of K become the values of relative brightness in different parts of a reflecting system and are plotted in Fig. 14.

From the data plotted in Fig. 14 it is possible to deduce the point at which it is desirable to change over from refractors to reflectors. The point is obviously that at which the brightness of refractors becomes less than the brightness of the corresponding reflectors and these values are:

ρ/ϕ	30′	1°	2°	3°	4°	5°	10°
Angle where refractors should cease	9°	13°	17°	19½°	21°	22°	29°

In practice, however, other considerations have to be taken into account and it is usual to cover about 30° with refractors. On occasion 40° has been covered with refractors, but it is definite that this is wrong.

The calculation of the beam given by a revolving lens for the case of a uniform spherical source can now be seen to be simply derived from the area of the refractors multiplied by the appropriate factor from Fig. 15, plus the area of reflectors multiplied by ·70, the whole to be multiplied by the brightness of the source.

(*f*) THE BEAM FROM A PETROLEUM VAPOUR BURNER MANTLE

Actually, however, uniform sources are not available and it becomes necessary to correct the results for variations from this idealised condition. As previously shown, any small element of surface gives a cone of light of angular area $\pi di'_T di'_R$ for the refractors and πdi^2 for the reflectors. The light across this small cone is uniform (actually it is an image of the light source distorted radially by chromatic aberration), falling sharply to zero at the edge. The values of di and di', however,

vary for all portions of the lens and thus at a large distance a point on a screen will be illuminated from a part only of the lens depending on the angular distance of the point from the axis. All points on the lens whose appropriate angle di'_T or di'_R is greater than this angle will contribute to the illumination or, alternatively, the shape of the beam at a large distance may be deduced from the values di'_T and di'_R for various parts of the lens.

The smallest external divergence angles are given by the reflectors, which will send light to the centre of the imaginary screen. The refractors above and below the centre will cover a somewhat larger angle vertically and horizontally. The horizontal portion of the refractors covers a greater angle still, considered horizontally. Thus the beam consists of a central circular portion of uniform intensity, surrounded by an annulus of diminishing intensity. The radius of the inner circle is given by the smallest value of di for the outer reflector and the radius of the extreme edge of the luminous patch by the maximum value of di'_R of the outer refractor.

Where the light source is not uniform it becomes necessary to compute values of di and the zone candle-power from the polar diagram, since both will vary in an uncertain way incapable of mathematical expression.

A series of tables has been computed for the petroleum vapour burner mantle measured as in Section III and the values of the expression (11) for various parts of the lens are given in Table XII. The whole beam power at a large distance is the sum of these contributions and the shape of the horizontal traverse depends on the horizontal spread of the light from each zone.

The beam given by the half lens consists of one-eighth from the vertical traverse above and below and one-quarter of the horizontal traverse. These values have to be totalled according to the horizontal angle covered by the beam. The values are plotted in curve *ABC* (Fig 16) and the experimental values are marked for comparison. Since the beam should be symmetrical, the left-hand and right-hand experimental values are both plotted to the right of the diagram. The agreement is so good that the fundamental soundness of the theory is confirmed. Curves *DE* and *FBC* (Fig. 16) represent the contributions to the total beam of the reflectors and refractors respectively.

As a petroleum vapour burner mantle approximates to a uniform spherical source, it is of interest to see how closely the beam power calculated from the results shown in Fig. 15 for such sources agrees with the measured value.

The brightness on the horizontal axis is 39·1 and the area of the mantle 29·1 sq. cm.

The radius of the equivalent circle is 30·4 mm. or $\rho/\phi = \cdot 0608$.

From Fig. 15 the value of the factor is ·496 or beam power from refractors = 2760* × 39·1 × ·496 = 53,500

and beam power from reflectors = 2400* × 39·1 × ·70 = 65,700

Total = 119,200

* Areas of refractors and reflectors.

TABLE XII. *Hartland Point light using petroleum vapour burner mantle*

Vertical traverse above axis $d\theta = 5° = \cdot0873$ radian

	Values of $2\pi\phi^2 \frac{\tan\theta}{\cos\theta} d\theta$	KB	$\frac{di_R}{di'}$	Expression (11) in thousands of candles	Horizontal angle covered di_T ($\cdot060 \cos\theta$)
			Refractors		
°					
0 ($\frac{1}{2}$ value)	0	—	—	—	·0600
5	120	31·3	·966	3·6	·0596
10	246	29·7	·928	6·78	·0590
15	380	28·2	·888	9·5	·0580
20	517	26·5	·842	11·5	·0564
25	691	25·4	·788	13·82	·0544
30	915	24·3	·731	16·25	·0520
35	1172	23·6	·667	18·45	·0491
40 ($\frac{1}{2}$ value)	750	23·0	·600	10·32	·0460
			Reflectors (from equation (16)) cover from 42°–55° approx.		
45	1942	20·7	—	40·3	·0423
50	2550	20·8	—	53	·0385
55 ($\frac{1}{2}$ value)	1715	21·0	—	36	·0344
			Vertical traverse below axis		
0	0	32·8	1·000	—	·0600
5	120	33·9	·966	3·9	·0596
10	246	35·0	·923	7·9	·0590
15	380	34·8	·871	11·5	·0580
20	517	34·6	·806	14·4	·0564
25	691	35·3	·737	18	·0544
30	915	35·9	·657	21·6	·0520
35	1172	36·6	·577	24·7	·0491
40	750	35·3	·485	12·8	·0460
45	1942	31·6	—	61·4	·0423
50	2550	29·9	—	76·25	·0385
55	1715	31·1	—	53·4	·0344
			Horizontal traverse across lens		
0	0	32·9	1·000	—	·0600
5	120	32·9	·973	3·8	·0617
10	246	32·9	·930	7·5	·0635
15	380	32·9	·890	11·1	·0652
20	517	32·9	·838	14·2	·0673
25	691	32·5	·782	17·6	·0696
30	915	32·1	·721	21·3	·0722
35	1172	31·3	·652	23·9	·0754
40	750	30·5	·582	13·3	·0792
45	1942	27·4	—	53·3	·0423
50	2550	27·4	—	69·8	·0385

This value is so near to the true result that for practical purposes it is sufficient to work out the value of ρ/ϕ for a given mantle and lens, using for r the equivalent radius deduced from the measured area of the mantle. From the value of ρ/ϕ so found, the appropriate value for the mean brightness of the refractors is taken from Fig. 15 and hence the beam candle-power is determined.

In the case of the Hartland Point apparatus, had refractors only covered 30° from the axis, the beam would have been

Refractors $39{\cdot}1 \times 1310 \times {\cdot}616 = 31{,}600$

Reflectors $39{\cdot}1 \times 3840 \times {\cdot}70 = 105{,}000$

Total = 136,600

or an increase over the existing arrangement of 17,400 candles or 14·6 per cent.

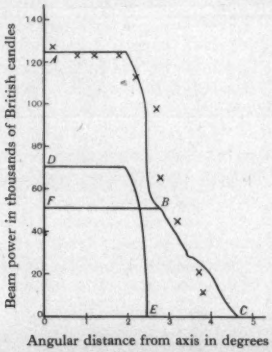


Fig. 16. Candle-power of beam, using a "Chance" 55 mm. autoform petroleum vapour burner (experimental results shown by crosses)

(*g*) The Beam from Fresnel Apparatus using a Cylindrical Strip as Source

The beam given by an electric filament lamp is the sum of the beams given by each separate filament. The positions of these small sources are fixed uniformly round the true focus of the apparatus and it is necessary to consider the beam from at least two positions:

(i) A filament at the focus;

(ii) A filament at a small angle away from the focus.

It may be said at once that the complete calculation of the beam is too complicated, but that an estimate of the maxima in the curve may be made.

A filament is taken as a cylindrical strip of length $2l$ and diameter $2t$ and is assumed to be placed vertically. The position is assumed to be focal. The flux of light on a zone between θ and $\theta + d\theta$ is given by

$$4lt \cos\theta B \,.\, 2\pi \sin\theta \, d\theta \qquad \ldots\ldots (18)$$

and the emergent light takes the form of a pyramidal cone of angular size

$$4\, di'_T \, di'_R \qquad \ldots\ldots (19).$$

Since the light source is unsymmetrical, the values of di_T and di_R will vary according to the position of the element considered. Thus for an element vertically above or below the centre,

$$di_T = \frac{t}{\phi}\cos\theta = di'_T,$$

$$di_R = \frac{l}{\phi}\cos^2\theta.$$

Hence the intensity is given by

$$\frac{4ltBK\phi\, 2\pi\cos\theta\sin\theta\, d\theta}{4t\cos\theta\, di'_R} = \frac{2l\pi\phi KB\sin\theta\, d\theta}{di'_R} \qquad \ldots.(20\text{ A}).$$

An element horizontally placed with reference to the axis will give an intensity

$$\frac{2\pi t\phi KB\sin\theta\, d\theta}{di'_R} \qquad \ldots.(20\text{ B}),$$

since

$$di_T = \frac{l}{\phi}\cos\theta = di'_R,$$

$$di_R = \frac{t}{\phi}\cos\theta.$$

There is little point in deducing the brightness of the lens in this case, as it obviously varies with the orientation as well as with the value of θ.

The results for one filament placed focally are given in Table XIII and the total for the half lens is obviously the mean of the two cases.

For the reflectors obviously $di_T = di'_T$ and $di_R = di'_R$ in each case or the intensity

$$= 2\pi\phi^2 KB\frac{\tan\theta}{\cos\theta}\, d\theta \text{ for vertical traverse} \qquad \ldots.(21\text{ A}),$$

and

$$2\pi\phi^2 KB\tan\theta\, d\theta \text{ for horizontal traverse} \qquad \ldots.(21\text{ B}).$$

TABLE XIII. *Hartland Point light using one filament placed focally*

Vertical traverse: Refractors $d\theta = 5° = \cdot0873$ radian; $t = \cdot1$ cm., $l = 1\cdot0$ cm.

θ	$2\pi l\phi\sin\theta\, d\theta$	KB	di'_R	Expression (20A) in thousands of candles	Angle covered di_T
°					
0	0	1050	·0200	0	·00200
5	2·39	1050	·0216	116	·00199
10	4·75	1050	·0232	215	·00197
15	7·08	1050	·0245	303	·00193
20	9·37	1050	·0261	377	·00188
25	11·55	1035	·0277	433	·00173
30	13·70	1023	·0295	475	·00164
40 ($\frac{1}{2}$ value)	8·83	976	·0345	250	·00153

Table XIII (*continued*)

Vertical traverse: Reflectors

θ	$2\pi\phi^2 \frac{\tan\theta}{\cos\theta} d\theta$	KB		Expression (21 A) in thousands of candles	Angle covered di_T
°					
45	1942	885	—	1700	·00141
50	2550	885	—	2230	·00128
55 ($\frac{1}{2}$ value)	1715	885	—	1500	·00115

Horizontal traverse: Refractors

θ	$2\pi t\phi \sin\theta d\theta$	KB	di'_R	Expression (20 B) in thousands of candles	di'_R
°					
0	0	1050	·0020	0	·0020
5	·24	1050	·0037	68	·0037
10	·47	1050	·0055	90	·0055
15	·71	1050	·0072	103	·0072
20	·94	1050	·0093	106	·0093
25	1·15	1035	·0116	102	·0116
30	1·37	1023	·0142	98	·0142
35	1·57	1000	·0174	91	·0174
40 ($\frac{1}{2}$ value)	·88	976	·0212	40	·0212

Reflectors

θ	$2\pi\phi^2 \tan\theta\, d\theta$	KB		Expression (21 B) in thousands of candles	di'_R
°					
45	1370	885	—	1210	·00141
50	1635	885	—	1445	·00128

The distribution to one side of the axis from this filament is obtained by summing the figures above according to the angle covered. The results are given in Table XIV.

The maximum value obtained is appreciably higher than the maximum found experimentally. This may be due to two reasons:

(i) No filament is exactly focal and this reduces the peak (see below).

(ii) In computing the brightness of the filament, the horizontal candle-power was used and this light was assumed to come from the ten filaments in a plane at right angles to the axis. If any of the filaments parallel to the axis contributed some light, as is indicated in Fig. 11, the value for the brightness would be too high. From Fig. 11 it appears that there is an area equivalent to nearly eleven filaments, thus giving a brightness of 1140 instead of 1250. The maximum would be in this case 2,630,000. This point cannot be settled with certainty without making a special lamp having only one grid.

TABLE XIV. *Distribution of candle-power from one filament*

Angle covered radian	Power of beam in thousands of candles
·00115	2860
·00128	2480
·00141	1570
·00153	840
·00164	780
·00173	650
·00181	540
·00188	425
·00193	330
·00197	250
·00199	203
·00200	174
·0037	174
·0055	157
·0072	140
·0093	110
·0116	85
·0142	55
·0174	30
·0212	10

(*h*) THE BEAM FROM AN EX-FOCAL FILAMENT

Consider the filament to be moved from the focus in a direction parallel to the lens face, i.e. normal to the axis. Then the elements vertically above and below the axis would give the same shaped beam as before, but with the axis of the beam at an angle to the axis of the lens equivalent to the angular movement of the filament. The portion of the lens horizontal from the centre, however, would give a beam spread out horizontally, since the angular displacement of the filament would be different for every value of θ. If α be the value of the angle given by the distance moved divided by the focal length, the angular movement considered from an element at a distance θ from the axis in a horizontal direction would be $\alpha \cos^2 \theta$. Thus the zone power from a given zone of the horizontal portion of the lens would be sent in a direction $\alpha \cos^2 \theta$ from the axis.

The values given in Table XV refer to the case of a filament at 1° (about the second filament in the grid) away from the focus.

These values are plotted in Fig. 17, where the summations have been made to give the distribution curve on one side of the axis for the beam from one filament at an angular distance of 1°.

Now the whole beam as measured is the summation of the beams from twenty filaments, ten being normal to the axis and ten parallel. A filament on a line normal to the axis gives a distribution similar to that shown in Fig. 17, but one coming between the two shown in the diagram would be superimposed on the secondary "hump" given by the next one. Thus the maximum for the second filament would be about 2,700,000, and as the axis was left each maximum would decrease, as is

TABLE XV. *Beam from prisms vertically above and below axis*

Angle covered each side of 1° radian	Beam power in thousands of candles
·00115	2000
·00128	1650
·00141	1090
·00153	660
·00164	603
·00173	480
·00181	380
·00188	250
·00193	160
·00197	80
·00199	30
·00200	0

Beam from prisms horizontally from axis

Zone beam power in thousands of candles	Position of middle of zonal beam relative to axis radian	Range on each side of middle radian
0	·0175	·0020
17	·0172	·0037
22	·0170	·0055
26	·0162	·0072
26	·0154	·0093
26	·0144	·0116
24	·0131	·0142
22	·0117	·0174
10	·0103	·0212
303	·0088	·00141
380	·0072	·00128

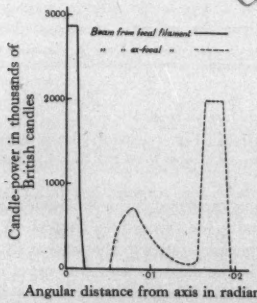


Fig. 17. Distribution of candle-power in the beam from an ex-focal filament

seen in the experimental curve (Fig. 3). The beam given by the filaments placed ex-focally on a line parallel to the axis would give a somewhat similar curve to that shown, but the maxima would come in intermediate positions.

The conclusion is that the maximum reading given by an electric lamp would correspond to the power given by a single filament placed focally, and that the additional filaments only give secondary maxima at a small angle from the axis. The brightness from which the power should be calculated is therefore the actual filament brightness and not, as usually adopted, the mean brightness of the grid derived from the candle-power divided by the area of the grid.

It remains to find how closely the beam power may be calculated on the assumption that the filament is a spherical source of equal projected area.

The area of a filament is ·40 sq. cm.

The radius of the equivalent circle is ·36 cm.

Hence $\rho/\phi = \cdot0072$, whence, from Fig. 15,

Relative mean brightness = ·247

The power from refractors = ·247 × 2760* × 1250† = **852,000**

The power from reflectors = ·70 × 2400* × 1250† = **2,100,000**

The values previously calculated are **840,000** and **2,020,000**, which are in fair agreement.

SUMMARY

Section I. (*a*) The distribution of light from the third order panel for Hartland Point was measured at different angles to the optical axis, using a "Chance" 55 mm. autoform petroleum vapour burner as the light source. The maximum value was about 125,000 candles.

(*b*) The measurements were repeated, using a 3 kW. lighthouse electric cruciform lamp. The maximum readings were about 2,000,000 candles. The most remarkable feature was the alternation of light and dark beams as the main beam was traversed.

Section II. The brightness of the lens relative to the brightness of the source was measured by screening off all except a small aperture. Corrections were applied to the results for the varying distance of the screen from the lens and photometer, and also for the fact that the beam consists of light and dark zones, whereas only light zones were measured. The brightness was found to fall off rapidly as the outer refractors were reached, but to be constant over the reflectors.

Section III. (*a*) Polar curves of the various illuminants were obtained vertically and horizontally.

(*b*) The variation of the candle-power of the petroleum vapour burner with oil pressure was investigated and a position of maximum efficiency found at 40 lbs. per sq. inch.

(*c*) The relative variation in candle-power of the 3 kW. and comparison lamps with voltage changes was measured and found to be negligibly small.

* Areas. † The brightness of the filament.

Section IV. A theoretical discussion of the beam from a Fresnel apparatus is given.

(*a*), (*b*) The divergence of a small pencil after refraction through a prism of known dispersive power is discussed.

(*c*) The intensity of the beam from refractors of the Fresnel type is calculated and the mean relative brightness of the whole lens is deduced.

(*d*) The transmission factor of the lenses was determined and the relative brightness of a refracting lens for different zones and light sources, assumed as uniform spheres, of varying angular diameter was found. The relative mean brightness for use in computing the total beam from refractors was found. The values agree exactly with the experimental values obtained in Section II.

(*e*) The intensity of the beam from reflectors of the Fresnel type was calculated. The relative brightness is shown to be constant over the angular range found in practical cases.

(*f*) The distribution of light in the beam from a petroleum vapour burner, using as a basis for calculation the polar curve previously found, was determined. The calculated figures agree well with the measured value obtained in Section I (*a*). It is shown that the petroleum vapour burner may be assumed as a uniform spherical source of the same projected area, and the beam calculated on this assumption from the tables of Section IV (*d*) agrees well with the measured value.

(*g*) The distribution of light given by single filaments placed focally is worked out. The maxima found are substantially above the experimental values of Section I (*b*), but reasons for this discrepancy are advanced.

(*h*) The distribution is calculated for an ex-focal filament and the general shape of the beam to be expected from an actual lamp indicated. It is shown that the assumption of a filament as a uniform sphere of the same projected area gives results in fair agreement with those calculated by the more cumbersome method.

The theory advanced is shown to give results in agreement with experiment. It is possible to compute the factor for the equation

$$\text{Beam power} = \text{factor} \times \text{area of lens} \times \text{brightness of source},$$

and curves of the appropriate values are given for all sizes of illuminant.

It follows from these results that the angle at which it becomes economical to change from refractors to reflectors is smaller the smaller the light source, and that in no case is it advisable to exceed 30° on each side of the axis with a refracting system.

The author's acknowledgments are due to the Directors of Messrs Chance Brothers and Co., Ltd., Glass Works, Smethwick, for permission to publish the above work, and to Messrs N. Hopkins and A. B. Lewis, who carried out all the experimental work.

DISCUSSION

Mr A. Brebner: I would like to congratulate Dr Hampton on his paper; it is the most interesting one on lighthouse optics which I have read for many years. Students of the subject might like to read a paper on "Powers of Lighthouse Lights by Calculation*," and a later one on "Power Formulae for Lighthouse Lights†," which describe a method differing somewhat from that given by Dr Hampton. Anyone who might read the former paper should beware of a few unfortunate misprints.

Mr J. P. Bowen: In thanking Dr Hampton for his paper, and Messrs Chance Brothers for granting facilities for the work, I should like to express my opinion of its value from the lighthouse engineer's point of view, as hitherto the method of computing the intensity of a beam from the panel of a lighthouse optic has been a matter of dispute amongst Lighthouse Authorities. Several attempts have been made to arrive at a standard formula, which it is recognised must be empirical owing to the imperfections in the glass of the various reflecting and refracting elements of which the panel is built up, the inequalities of the burner or source of light, and the losses which are experienced as the rays of light pass through the glass media of the optic and lantern, but so far no universal system is employed. It is, therefore, to be hoped that members of the Optical Society will thoroughly examine Dr Hampton's paper in all its details, and criticise to the full the assumptions and theories that he has put forward, so that a correct basis on which to construct the embryo of a formula may be found.

Mr J. R. Wharton: Mr Bowen has been good enough to thank Messrs Chance Brothers in connection with Dr Hampton's paper and I should like to take this opportunity of expressing our thanks to him in return. As has been pointed out, it was at Mr Bowen's suggestion that the investigation made by Dr Hampton was carried out and the Corporation of Trinity House have assisted us very materially, both by the loan of apparatus and otherwise.

I should like to add my request to that already made by Mr Bowen, that the Optical Society will carefully consider the results obtained in Dr Hampton's paper. It would be of great value to us as manufacturers to have a basis for the calculation of the intensity of lighthouse optical apparatus agreed upon internationally.

Dr R. S. Clay: As a physicist I am interested in the powerful light sources mentioned by Dr Hampton. I assume that the electric light referred to is a metal filament coiled in two vertical planes at right angles to one another in a gas-filled bulb, and that the ghost is the image formed by reflection in the hollow surface of the glass. If so, it should be possible by a suitable adjustment of the position of the filament in relation to this surface to obtain the image superposed on the filament if desired.

Why does the surface intensity of the mantle of the vapour lamp decrease towards the top of the mantle? The hottest part of the flame, which I suppose is

* *Proc. Inst. Civil Eng.* **122** 1894–5 (300). † *Ibid.* **199** 1914–15 (388).

where the mantle should be situated, is I believe on a surface where the combustion is most intense and this is where the flame is fed with some admixture with the surrounding air. Is this not as hot at the top as it is lower down? Also with a mantle of given dimensions I would expect to obtain the maximum illumination when the pressure of the vapour is adjusted so that the surface of greatest intensity of combustion coincides with the surface of the mantle. Might this be the reason why Dr Hampton obtained his maximum at a comparatively low pressure if he happened to be working with a smaller mantle?

Mr J. F. Ramsbotham (communicated): I regret that time did not permit me to take part in the discussion, as there are several points which I would like to have raised.

The Corporation of Trinity House are certainly not only to be congratulated, but also to be thanked for enabling Messrs Chance Brothers to carry out these very important and interesting experiments.

The luminous power of lights is of the greatest importance to mariners, and the International Maritime Conference of Washington recommended the adoption of uniform rules in order to define the luminous power and visibility of lights according to the state of the atmosphere and their geographical range in relation to the focal plane and the height of the eye of the observer, as published in light lists. The units adopted are not the same in different countries, but as their relation is known, no serious inconvenience results.

The most important thing is the definition and practical measurement of the power related to the atmosphere. No doubt Messrs Chance Brothers observed the atmospheric conditions when conducting their experiments in Birmingham.

I agree with Mr Bowen about the international importance of these experiments, but would suggest going a step further and incorporating in the experiments lights illuminated by acetylene, embracing automatic flashing lights with fixed lenses and a flasher, and lights of a higher candle-power, using autoform mantles, and also illuminated by acetylene.

In relation to these, I would like to mention an interesting experience in connection with alterations to a light at Low Head, Tasmania, Australia. It had been decided to substitute a vaporised kerosene, third order, Chance Brothers apparatus for the existing light, the new light consisting of a group flashing white light showing three flashes every fifteen seconds, viz.

Flash	*Eclipse*	*Flash*	*Eclipse*	*Flash*	*Eclipse*
$\frac{1}{3}$	2 secs.	$\frac{1}{3}$	2 secs.	$\frac{1}{3}$	10 secs.

The light was 63,000 C.P., and the illuminant was vaporised kerosene with a collodion mantle, with a visibility of 18 miles. During the alterations a temporary light was used, consisting of a flasher and a drum lens, giving 5000 C.P., the illuminant being acetylene gas, and the character being a single flashing white light with one flash every five seconds, thus:

Flash	*Eclipse*
$\frac{1}{2}$ sec.	$4\frac{1}{2}$ secs.

After the temporary light had been removed, and the new light established for

some time, Master Mariners using that light came to the writer and said that they preferred the temporary light (with the lower candle-power). It must be remembered that Low Head is one of the few places in Australia where fogs are experienced. Perhaps the author can offer some explanation for the preferment.

As regards the use of electric lights, I can only say that electric light as an illuminant was tried in Australia in 1881, being discarded in 1912, an incandescent vaporised kerosene illuminant being substituted in its place. The experiment was never repeated, as the first cost was too heavy, it having been found necessary to have duplicate plants, with the exception of the optical parts; and further, it was found that the resulting candle-power was altogether in excess of requirements, notwithstanding that the light was a landfall light: in addition to which the running costs were altogether excessive when compared with the vaporised kerosene, and the State Government scrapped the apparatus for reasons of economy, as already mentioned.

I am aware that considerable improvements have taken place with regard to both the generating of electricity and the illuminant since 1881; but I can only say that, when I was appointed Director of Lighthouses to the Commonwealth of Australia in 1913, the question of using electricity was carefully considered, and was turned down solely on the score of expense.

It would add considerably to the value of the paper if the author could give the relative first costs and the subsequent running costs of

(*a*) a standard petroleum vapour burner light;

(*b*) a 3 kW. lighthouse cruciform lamp light, the latter being run in duplicate.

I wish to join in the congratulations to the author on his paper, as there is very little literature on the subject, the nearest approach to it being the book by the late Mons. C. Ribière, Engineer-in-Chief to the Lighthouse Service, France.

Dr Hampton: I am obliged to Mr Brebner for his congratulatory remarks, since the subject of the paper is, of course, one to which he has paid particular attention. I had read his papers previously and the present investigation might be considered as an attempt to find the value of the factor employed in the formula he proposed.

Mr Bowen has been particularly kind in the interest he has taken in the work from its commencement, and I trust that his request that the paper should be critically examined will be followed up by the members of the Society.

Dr Clay's assumption as to the shape of the filament is correct, and since the ghost is, as he says, formed by reflection on the inner face of the bulb, I see no reason at all why the image should not be superposed on the filament. From many points of view this would be an advantage, but Mr Bowen has suggested that the present position of the ghost, which is slightly above the filament itself, offers an advantage in that it throws a relatively weak beam somewhat inshore of the main beam.

With regard to the question of the brightness of the mantle decreasing towards the top, I have already mentioned in the paper two reasons for this, and reference to the photographs reproduced makes it, I think, fairly clear. Viewed from above

the mantle appears less bright because of its semi-transparency which allows the black grid at the top of the burner to be seen through the mesh. When viewed from below, the hot surface at the back of the mantle is seen through the mesh at the front, thus to a large extent filling up the gaps in the single surface. It is, I think, generally accepted that a Bunsen flame of the Méker variety is hottest just above the blue cones, and this also causes the mantle to be brighter near the bottom. The mantles used by these burners are of the autoform variety which means they are flexible when put on to the flame, and so they do to a large extent follow the shape of the flame itself.

The measurements of the pressure in the vaporiser chamber referred only to the particular mantle used, and doubtless different results would be obtained with different burners and different mantles.

The question raised by Mr Ramsbotham as to the atmospheric conditions obtaining while the experiments were in progress is answered by the facts that first, the distances used were relatively small, and secondly, the nights during which the measurements were carried out were quite clear although without a moon. I do not think the results are subject to any appreciable correction in this respect.

It would certainly be interesting to carry out further experiments on different light sources, but of course the general theory advanced should hold good for any light source provided the data necessary are taken from a polar diagram. It is quite probable that to a sufficiently close approximation even an acetylene flame could be considered as a spherical source.

I am afraid it is impossible for me to offer any explanation for the preferment by seamen for the lower power of light. If the mariners themselves do not offer any explanation it would be presumptuous for anyone not having full access to all the particulars to offer an opinion.

With regard to Mr Ramsbotham's remarks and queries on the use of electric light, I regret that they do not come within the scope of my investigations, as they deal with the question of costs. I understand the position has been entirely changed during the last few years by the development of special gas-filled lamps suitable for lighthouse purposes.

Since the paper was written further measurements have been carried out on a first order panel for Castle Island. The illuminants in this case were, as before, a Chance 55 mm. petroleum burner mantle and a 3 kW. G.E.C. cruciform lamp. The measurements were not carried out in the same detail as those for Hartland Point, but the corrected measured values and the values calculated on the assumption that the burner was a spherical source of the same projected area, are as follows:

Illuminant used	*Measured candle-power*	*Calculated candle-power*
Petroleum vapour burner	477,600	524,000
3 kW. cruciform lamp	12,030,000	12,220,000

The agreement between measured and calculated values is again satisfactory.

As some readers may not be familiar with the appearance of lighthouse lenses, I have added a photograph (Fig. 18) of the complete lens for Hartland Point.

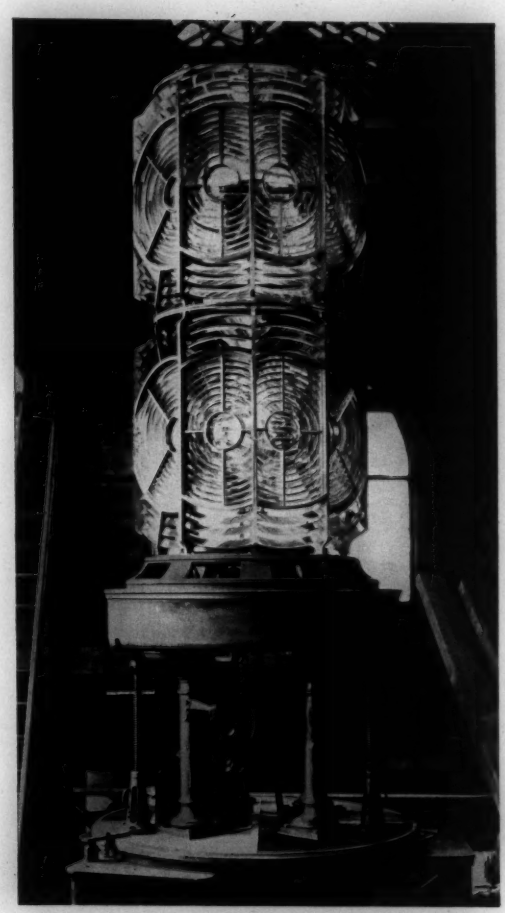


Fig. 18. Complete lens for Hartland Point

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THE "ABSOLUTE" HARTMANN TEST

By R. KINGSLAKE, M.Sc., A.R.C.S., D.I.C., F.R.A.S.

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ABSTRACT. According to the ordinary methods of performing the Hartmann test, the assumption is made that all the rays cross the optical axis. As a matter of fact, this is very far from the truth, especially in the case of the large astronomical objectives and mirrors to which the Hartmann test is most generally applied. In the present paper a brief description is given of a modified test in which each "ray" is treated on its own merits and its absolute deviation from the ideal path is determined in both magnitude and direction. This method incidentally leads at once to the well-known criterion "T" of Hartmann, which is now generally adopted in the comparison of large telescopes.

Introduction

In most of the instances where the Hartmann test has so far been applied, a grave assumption has been made, namely, that all the rays leaving the lens or mirror cross the axis at some point. This is probably justifiable in the case of small systems which have been made to test plate, but in any lens of greater aperture than about 4 inches irregularities in the glass make figuring necessary, and then the finished product may easily be such that the emerging rays never actually intersect the axis at all but pass a little to one side of it. With large concave mirrors which must of necessity be figured, it is even more probable that but few of the reflected rays will exactly intersect the axis of the mirror.

When the Hartmann test is applied to large astronomical instruments it is usual* to employ a diaphragm drilled with holes arranged so that four equally spaced holes shall fall on each zone under examination. It is then assumed that the narrow pencils of light through these holes will all cross the optical axis, and the longitudinal distance of this crossing point is deduced from measures of the distances apart of the various pairs of dots on the photographs taken within and beyond the focus. The average of these supposed intersections of the four rays with the axis is regarded as the "focus" of the zone, and by using holes on 12 to 15 of such zones, very fair zonal aberration curves can be constructed. Differences between the foci of the individual holes in a zone are attributed to astigmatism.

Now in any good instrument of large size it is not safe to assume that the departures of the rays from their ideal tracks will follow any precise mathematical law. In bad cases of under-corrected spherical aberration, for example, it is likely

* See P. Fox, on the Yerkes telescope, *Astrophys. Journ.* **27** 1908 (237); J. Hartmann, on the Potsdam 80 cm. objective, *Z. f. Instrk.* **29** 1909 (217); and G. Shajn, on the Simeis 40 inch reflector, *Bull. Pulkovo Obs.* **10** 1926 (450).

that a law of the form $a = A + BY^2 + CY^4 + \ldots$, where a is the aberration and Y the semi-aperture, will be found to fit the observed aberration fairly closely; but in the case of a lens or mirror which is very nearly perfect the presence of "theoretical" or "mathematical" aberration is unlikely, and it will all be "technical" aberration which has arisen in the manufacture of the glass or in the figuring of the surfaces. In this case each ray-track must be considered on its own merits, and no averaging is permissible. A Hartmann test on these lines may be called an "absolute" Hartmann test, and a few notes on its performance may be of interest.

General Considerations

The first point to notice in considering such a test is that both the inside-focus and outside-focus patterns must be obtained on one plate, and that the plate must be moved strictly parallel to itself between the two exposures. It is advisable that one row of holes across the lens should be taken as the X axis of the system, and that the lower edge of the photographic plate should be always parallel to that row of holes. This gives us a definite reference axis on the plate, and the positions of the dots can all be measured relative to this axis. If by any chance the plate is allowed to rotate in its own plane between the two exposures, it will appear subsequently as if the entire cone of rays were twisted, and hence such a rotation must not on any account be permitted.

That diameter of the lens which is perpendicular to the chosen X axis can be regarded as the Y axis, with the usual sign convention as regards coordinates when the lens is viewed from the *object* side. The advantage of this is that, when the films of the plates are examined, the coordinates will again have the usual geometrical sign relation.

The process of computation is now quite straightforward, only it must be performed twice, once for the plan view of the system (the X coordinates), and again for the elevation view (the Y coordinates). The absolute displacement of each ray can then be determined in both magnitude and direction.

Choice of Diaphragm

Any arrangement of holes in the diaphragm may be used, and the holes need not be positioned with great care. The only condition which must be fulfilled is that the holes are distributed as equally as possible all over the lens aperture. There is no need to confine the holes to a limited number of zones or to any specified diameters of the lens, although this would be an advantage in the interpretation of the results. For convenience in the measurement and computation, a series of equidistant rows of holes across the aperture would be as good as any, provided that there is some ready means of identifying each of the dots on the plates. A radial formation is certainly valuable from this point of view, but a good many extra holes should be placed between the outer ends of the radii in order that as little of the lens as possible is excluded from the test. As this test is mainly to examine the quality of the figure, as many "samples" of the lens should be taken as possible.

Whatever arrangement of holes is employed, it is necessary to assign a number, or a letter and number, to each separate hole. All the measurements and computations are done hole by hole, and the results will be tabulated for the separate holes in order. The actual coordinates of the holes in the diaphragm need not be tabulated as they do not enter into the calculations.

A *central* hole is absolutely necessary as this provides the "ray" with which all the other "rays" are compared. An odd eccentric hole should also be made to locate the "top" of the pattern when the plate is being measured.

Experimental Considerations

The method of performing the experiment is as usual*. A monochromatic source is preferable, but if a real star is to be used, Plaskett† recommends the use of a red star with a lantern plate, as in this case the actinic part of the star's light is confined to a very short range of wave-lengths, and is thus sensibly monochromatic. If white light is used, the dots on the plates become spread out into short radial spectra. When testing a mirror, white light may of course be used.

The plate-holder must be mounted on a slide by which the plate can be moved exactly parallel to itself along the axis from one Hartmann plane to the other without any rotation whatever; and at the same time the plate must be moved across the cone of rays, the direction of movement being exactly parallel to the X axis of the plate. This is conveniently attained by taking the lower edge of the photographic plate as the X axis, and simply sliding the plate along on its edge between the exposures. Alternatively, the plate may be attached rigidly to a sliding fitting, such as a simple geometric V-and-cylinder slide.

The two patterns should fall as close together as possible on the plate to simplify the measurement. It is also a convenience, especially if a single-slide measuring microscope is to be used, to rule a line between the patterns exactly square to the base of the plate. This line will represent the Y axis, and may be scratched on the film with a needle after the plate has been developed and dried.

The patterns should be small, but not so small that the dots run together. Very close dots frequently have diffraction fringes between them, which must be avoided as they tend to displace the dots from their true positions.

Measurement of the Plate

The finished plate will appear as shown in Fig. 1. The best form of measuring instrument is a small cross-slide travelling microscope with two perpendicular micrometer screws. The travel need only be about an inch each way, although it would be an advantage if one of the slides were long enough to cover both patterns at one setting. Failing this the X and Y coordinates of each dot on each pattern

* Hartmann's original paper may be found in *Z. f. Instrk.* **24** 1904 (1). See also R. Kingslake, *Proc. Opt. Convention*, 1926 (839).

† Plaskett, *Astrophys. Journ.* **25** 1907 (195).

can be measured separately with an ordinary single-slide plate-measuring microscope such as is used in spectroscopy. Whatever process is adopted, the scratched line will represent $X = 0$ and the bottom edge of the plate will represent $Y = 0$. The cross-slide microscope has the advantage that each dot need only be set once, and the two coordinates can be read off simultaneously.

In the following discussion, all measures on the inner photograph will have the suffix 1 and on the outer photograph the suffix 2. The coordinates of the central dots are denoted by small x and y, capital letters being used for the other dots.

Great care must be taken not to confuse the measures of the two plates. The inside pattern appears erect and the orientation will be similar to that of the original diaphragm; but the outside pattern will be inverted.

The measurements should be tabulated, and the coordinates of each dot relative to the central dot calculated, as follows:

Hole	X_1	X_2	Y_1	Y_2	l_1 $(X_1 - x_1)$	l_2 $(X_2 - x_2)$	h_1 $(Y_1 - y_1)$	h_2 $(Y_2 - y_2)$
centre	x_1	x_2	y_1	y_2	—	—	—	—

The greatest care must be taken in all this work, especially in regard to signs and possible confusion of the dots.

Fig. 1. A typical Hartmann plate

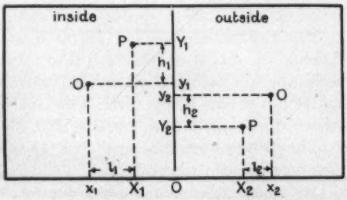

Fig. 2. The notation of the plate measurements

The Reduction of the Measurements

Suppose the diaphragm contained only the central hole O and one outer hole P (Fig. 2). We can now imagine this system as seen in plan and in side elevation (Fig. 3). For the particular ray under discussion l_1 and h_1 are both positive, while l_2 and h_2 are negative. The formulae can now be deduced on the basis of this sign convention.

It is the aim of the experiment to determine the two component lateral displacements (ξ, η) of our chosen ray from the central ray, the displacements being

located in a chosen plane at distance c beyond the inner plate. The plane c is the plane in which the cone of rays has the smallest overall size, and it is the same as the plane which Hartmann calls "the plane F_0" in the determination of the criterion "T" (see below). But until the calculation is complete, it will not be known precisely where the cone is narrowest, so it is better to chose a value of c as near as possible to the smallest part of the cone which can easily be determined with the help of a piece of ground glass.

The value of c being decided, and the distance d between the two photograph planes measured, we can at once deduce ξ and η for each ray by the formulae:

$$\xi = l_1 - \frac{c}{d}(l_1 - l_2), \qquad \eta = h_1 - \frac{c}{d}(h_1 - h_2).$$

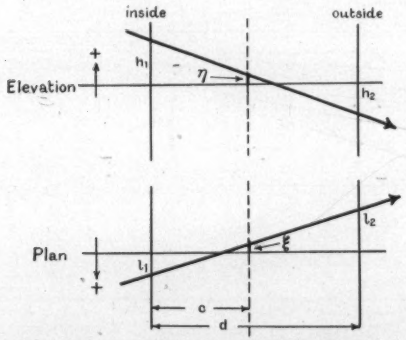

Fig. 3. Path of a typical ray between the Hartmann planes

In order to confirm whether this coincides with the smallest possible diameter of the cone, it is now advisable to compute a second series of values of ξ and η for a slightly different value of c. The plan and elevation views of the part of the cone between these two planes can then be drawn out to a large scale by plotting the ξ and η values of each ray and joining them up with straight lines. By comparing the two diagrams, one showing the plan view (ξ) and the other the elevation (η), it is seen at once what value of c would give the actual disc of least confusion. To save a third calculation for this best focus, the respective values for ξ and η for the various rays can be simply measured off from these drawings and tabulated.

In an actual experiment with a 6 inch objective and a fairly near monochromatic artificial star, the separation of the plates was 40 cm., and 30 holes were used, distributed as equally as possible over the aperture. The coordinates X and Y were tabulated and the values of l and h deduced. As a first guess $c = 20{\cdot}3$ was tried, and a table of the corresponding values of ξ and η was drawn up. For a second attempt, c was changed to 21·0 cm., and the values of ξ and η were again determined. On

plotting these figures and joining up corresponding points, the plan and elevation of the ray bundle were found (Fig. 4).

The narrowest part of the bundle was located at $c = 20{\cdot}74$ cm. and the final ξ and η values were read off for this plane.

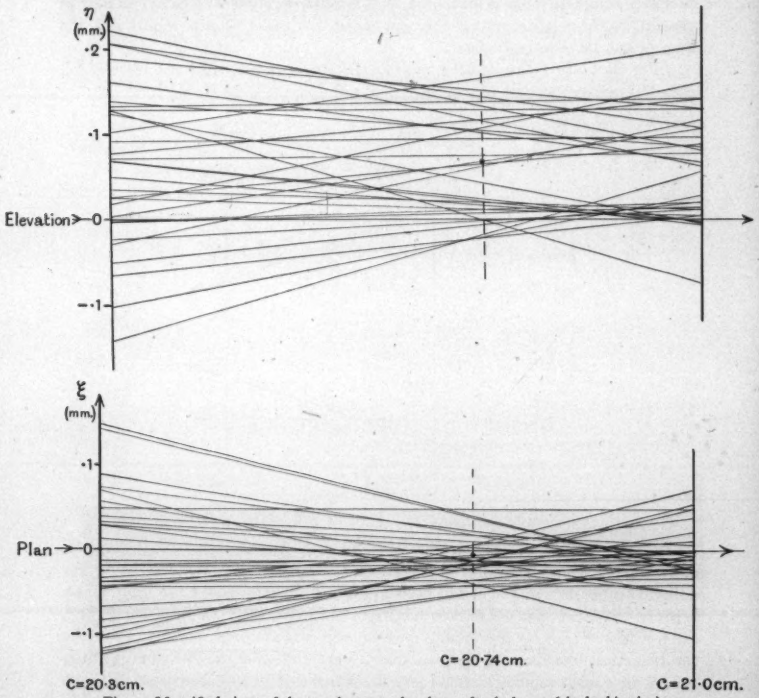

Fig. 4. Magnified views of the rays between the chosen focal planes (6 inch objective)

Interpretation of the Results

As mentioned above, it is best to express these displacements as angular deviations of the rays from their ideal tracks. To do this, we assume that the "ideal tracks" of all the rays would pass through the centre of the pencil in the plane c where the beam is narrowest. In practice this point will not lie exactly on the

central ray, as it is just as likely for the central ray to be wrong as for any of the other rays. So when the ξ and η in the finally chosen plane are read off, it is best to measure them from the centre of the bundle and not from the crossing point of the central ray. These final ξ's and η's are here denoted by heavy type.

Then the component angular deviations of each ray from its ideal track are ξ/f and η/f, where f is the focal length measured in the same units as ξ and η; or, in seconds of arc, the components are

$$\alpha = 206260\ \xi/f, \qquad \beta = 206260\ \eta/f.$$

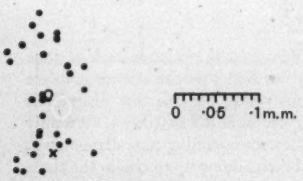

Fig. 5. Cross-section of pencil, in the plane $c=20{\cdot}74$ cm.

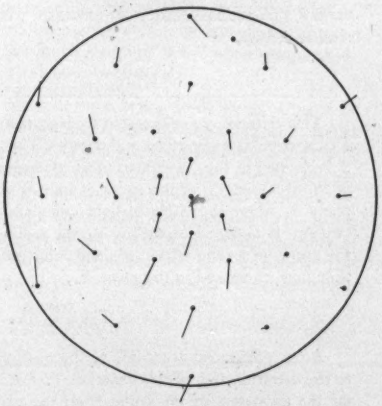

Fig. 6. Diagram showing the angular displacements of the separate "rays" (6 inch objective)

Then the whole deviation of the ray is of course equal to $\sqrt{\alpha^2+\beta^2}$. The magnitudes and directions of the ray deviations may however be determined graphically by plotting the ξ and η for each ray, and drawing the vectors from the centre of the pencil to the plotted points. These vectors give the required deviation in both magnitude and direction.

In the example of the 6 inch objective described above, the vector diagram was as shown in Fig. 5, the central ray being marked x and the centre of the pencil o.

Finally the ray deviation can be shown on a diagram of the actual holes in the Hartmann diaphragm, by a little line or arrow showing the displacement of that particular ray in both magnitude and direction (Fig. 6).

Such a diagram pictures the state of figure of the entire lens. Only those rays which have their arrows pointing radially towards or away from the central ray will actually intersect it, and it is obvious from this diagram how unjustifiable it is to assume that *all* the rays cross it!

Hartmann obtained a diagram of this form for the Potsdam 80 cm. objective*, but the method is not by any means as generally known as it deserves to be.

If the lens is not very good, and regular spherical aberration is present, it will be apparent in this diagram by all the arrows on each zone pointing radially outward or inward by about the same amount. Coma will lead to rings of similar radial arrows, but not concentric. Astigmatism will give an elliptic shape to the sets of similar arrows.

The Hartmann Criterion

This criterion was suggested by Hartmann† as a means of comparing the performance of large objectives and mirrors. The "Criterion" or "Technical Constant" T is defined as the "weighted mean diameter (expressed in hundred-thousandths of the focal length) of the cones of light from the various zones, measured in the plane F_0 where the circle of light containing all the pencils is the smallest. The 'weight' is assigned according to the radius of the zone." Hence, writing r for the radius of a zone of the lens and δ for the diameter of the cone of rays leaving that zone, measured in the plane F_0,

$$T = \frac{100000}{F_0} \cdot \frac{\Sigma r\delta}{\Sigma r}.$$

A test performed according to the method described above lends itself at once to the determination of this criterion T. For by our Fig. 5 we can at once measure off the diameters of the cones from the various zones, because that diagram is already worked out for the plane F_0 where the pencil is the narrowest. We only have to draw up a table of radii of zones r and the corresponding cone diameters δ, thence deducing the respective products $r\delta$, and then adding up to obtain the sums Σr and $\Sigma r\delta$. F_0 may be measured sufficiently nearly with a ruler. In general, however, the cross-sections of the cones of rays will not be circular. Hartmann does not specify how to treat this case, but it would appear reasonable to read off from Fig. 6 the actual linear displacements of the rays in each zone, measured in the plane F_0, and to take twice the average of these displacements as the mean diameter of the cone.

* J. Hartmann, *Z. f. Instrk.* **29** 1909 (217).

† J. Hartmann, *Z. f. Instrk.* **24** 1904 (46); P. Fox, *Astrophys. Journ.* **27** 1908 (237).

DISCUSSION

Prof. L. C. Martin: I take it that the original feature of this paper is the reference of all intercepts to the central ray; it seems to be simple and satisfactory. On looking at the "Fokogramme" photographs made by Hartmann for the Potsdam objective one cannot help wondering whether, if all the holes in the Hartmann diaphragm were shifted very slightly, the resulting vector diagram would bear the slightest resemblance to that originally obtained. The irregularities seem so diversified. In view of the difficulty in subjecting an objective of this kind to interferometer tests, is there no way of making the knife-edge test quantitative?

It is a small point, but one does get much sharper "dots" when using monochromatic light in testing a mirror. The reason is obvious.

Mr Kingslake: Undoubtedly the Hartmann test is limited to the actual parts of the lens or mirror which happen to fall under the holes in the diaphragm, and it is likely that a slight shift of the diaphragm would give a different vector diagram. But in any actual lens the local fluctuations from one point to a closely adjacent point would probably be far less than the general trend over several inches of the lens aperture. There is a great need for a quantitative test analogous to the interferometer test, which will give a measure of the errors of the whole aperture at once, with no limitation to the size of the lens under test. The knife-edge test gives a picture of the errors, which can be very useful if taken in conjunction with the Hartmann test.

SOME REMARKS ON AN EXTENSION OF THE OPTICAL COSINE LAW

By M. HERZBERGER, Ph.D.

(Messrs Ernst Leitz, Wetzlar.)

[Communicated by Mr T. Smith.]

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ABSTRACT. In this note reference is made to an extension of the optical cosine law, a full description of which will appear in the *Zeitschrift für Physik*.

The remarkable papers on the optical cosine law which have been published by Mr T. Smith in these *Transactions* have encouraged opticians in all countries to occupy themselves with the problems of geometrical optics. I wish in this note to refer to an extension of the optical cosine law about which Dr Lihotzky and I have written a paper which will be published in the *Zeitschrift für Physik*.

The problem we wished to deal with was how to get a uniform image of a finitely extended surface by an axially symmetrical optical system. In general, the rays coming from a point P of the object space form, after leaving the system, a pencil of rays having a *plane* of symmetry, i.e. the plane through the point P and the axis. If P lies on the axis, all planes through the axis are planes of symmetry and the pencil of rays leaving the system is axially symmetrical. The axis of the system is the axis of rotation of that pencil.

We say that a surface has a *uniform* image or "homöoplanatic" image if to every point of the surface there corresponds a pencil of rays that is axially symmetrical. It is easily seen that a *sharp* image, where to each point there corresponds a *point* (we call such an image an "aplanatic" image of the surface), is a very special case of the problem with which we are dealing.

Let us suppose that we have a "homöoplanatic" image of a surface. The rays which come from a fixed point P of the object space all meet, after leaving the system, a special ray, the axis of rotation of the pencil. We call that ray the principal ray of the pencil. The caustic surfaces enveloped by this two-dimensional manifold of rays degenerate into a portion of that principal ray and a surface which is rotationally symmetrical with respect to it. The principal ray is the only ray on which there is no astigmatism; it meets the caustic surface in a cusp.

Assuming that the rays coming from the point P touch an axially symmetrical

caustic after leaving the system, what conditions must be fulfilled by these rays in order that an element of surface round P will have a homöoplanatic image? All points lying on the same circle round the axis as P have an image which is obtained by rotating the image caustic round the axis. A sagittal line element (sagittal will mean perpendicular to the meridian plane, the plane through the point and the axis) will have a uniform image without a new condition. The axes of rotation of the point and the neighbouring sagittal point meet on the axis, forming a part of a cone. The apex of the cone we call the sagittal pupil point B'_s (Fig. 1). Let ϵ_s and ϵ'_s be the angles between rays in the object and image spaces respectively and the sagittal direction, k'_s the distance between B'_s and the cusp S

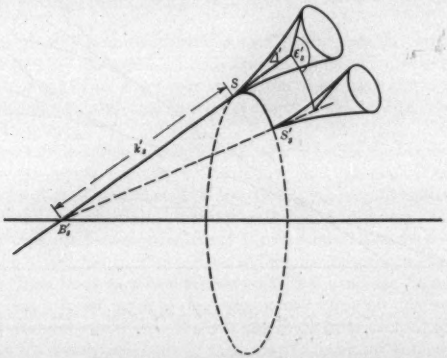

Fig. 1. The caustic with the cusp at S'_s is obtained by an infinitesimal rotation of the caustic with the cusp at S round the axis. SS'_s is a sagittal line element.

of the caustic, and $k'_s + \Delta'$ the distance to the point where any given ray meets the principal ray. We then obtain the following equation:

$$n'(k'_s + \Delta')\beta'_{s0}\cos\epsilon'_s - nk'_s\cos\epsilon_s = 0 \qquad \ldots\ldots(1),$$

where n, n' are the refractive indices of the object and image spaces respectively and β'_{s0} is the magnification in the sagittal direction measured on the principal ray. Equation (1) is a necessary and sufficient condition that P, and consequently all points of the circle through P round the axis, forms an axially symmetrical image. In order that an element of surface may have a homöoplanatic image, still another condition must be fulfilled, which gives the uniform image of a line element in the meridian plane. Also the neighbouring point in the meridian plane must have an axially symmetrical image caustic. The principal rays which belong to these two neighbouring points meet in a point of the meridian plane which we call the meridional pupil point B'_m (Fig. 2). Let k'_m be the distance of B'_m from the cusp S' of the caustic, $k'_m + \Delta'$ that from the point where some given ray meets the

principal ray, ϵ_m and ϵ'_m the angles between the rays in the object and image spaces respectively and the direction in which the cusp of the caustic advances in the meridian plane, and ϵ_{m0} and ϵ'_{m0} the same angles for the principal rays. A necessary condition is then

$$n'(k'_m + \Delta')\beta'_{m0}(\cos\epsilon'_m - \cos\epsilon'_{m0}) = nk'_m(\cos\epsilon_m - \cos\epsilon_{m0}) \quad \ldots\ldots(2),$$

where β'_{m0} is the meridional magnification measured on the principal ray.

Conditions (1) and (2) contain the well-known cosine law of Bruns if we put $\Delta' = 0$, i.e. if there is a perfect (aplanatic) image. They contain a special case of the

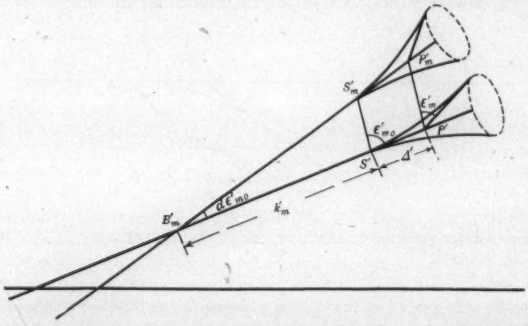

Fig. 2. The caustic with the cusp at S'_m is the neighbouring caustic to that with the cusp at S' in the meridian plane. $S'S'_m$ is a meridional line element. $P'P'_m$ is parallel to $S'S'_m$.

cosine law of T. Smith if we make $k'_m + \Delta' = k'_m = \infty$ (the pupil must be at infinity). If we take P on the axis,

$$\epsilon'_{m0} = \epsilon_{m0} = 90°, \quad \epsilon'_m = 90° - u', \quad \epsilon_m = 90° - u, \quad \beta_{m0} = \beta_{0s} = \beta, \quad k'_s = k'_m,$$

and we easily obtain

$$n'(k' + \Delta')\beta'_0 \sin u' = nk' \sin u \quad \ldots\ldots(3),$$

which is the isoplanatic condition of Staeble and Lihotzky, the condition that there is no coma near the axis.

If conditions (1) and (2) are fulfilled for a point P we are able to construct all caustics in the neighbourhood of P. I will show here how to construct the caustic which belongs to the neighbouring point in the meridian plane. As all caustics are axially symmetrical, we need only look at the rays in the meridian plane. The cusp of the caustic in the image space may change from the point S' (Fig. 2) on the first principal ray to the point S'_m on the next principal ray. Let the angle between the principal rays be $d\epsilon'_{m0}$. We say that a ray of the first caustic corresponds to a ray of the second caustic, if they meet the principal rays in two points P', P'_m such

that $P'P'_m$ is parallel to $S'S'_m$. Rays which correspond to one another meet in a point A'; let $A'P' = x'$. If the homöoplanatic condition has been fulfilled we find

$$x' = (k' + \Delta') \frac{\sin(\epsilon'_m - \epsilon'_{m0})}{\cos\epsilon'_m - \cos\epsilon'_{m0}} \sin\epsilon'_m \qquad \ldots\ldots(4),$$

and

$$\frac{d\epsilon'_m}{d\epsilon'_{m0}} = \frac{\cos\epsilon'_m - \cos\epsilon'_{m0}}{\sin\epsilon'_{m0}\sin(\epsilon'_m - \epsilon'_{m0})} \qquad \ldots\ldots(5).$$

By means of (4) and (5) we are able to construct the neighbouring caustic. On integrating (5) we obtain

$$\sin^2\frac{\epsilon'_m - \epsilon'_{m0}}{2} \sin\frac{\epsilon'_{m0}}{2} = \sin^2\left(\frac{\epsilon'_m - \epsilon'_{m0}}{2}\right)_{\text{axis}} \qquad \ldots\ldots(6).$$

From formula (6) we obtain the caustic at each point, from the form of the axial caustic, if we suppose that there is a homöoplanatic image of a finite surface.

We see from the form of (6) that all caustics must have the same degree of spherical correction and the same number of cusps, in short that all the topological qualities are maintained.

There remain only a few words to be said as to whether the homöoplanatic law (and also the isoplanatic law, the aplanatic law, and the general cosine law of T. Smith) is a sufficient condition. The answer is in the negative, as may easily be seen. The condition that a line element should have a uniform image must not involve the preferential treatment of one of its two points, while the cosine law does so. We obtain a *sufficient* condition if we differentiate equations (4) and (5) in a suitable way. This leads to a new theorem which we call the "Correspondence Theorem." This theorem gives us, for each point on any ray, the corresponding image points in the image space. We obtain now the image caustics for all points of the space if we merely suppose that there may be one finite surface which has a homöoplanatic image. This latter theorem will be explained and demonstrated on another occasion.

REVIEW

Department of Scientific and Industrial Research. Illumination Research, Technical Papers (London: H.M. Stationery Office, 1926–7):

No. 1. *The Terminology of Illumination and Vision.* Pp. iv + 21; 7 Figs. Price 9*d.* net.

No. 2. *The Transmission Factor of Commercial Window Glasses.* By A. K. TAYLOR, A.C.G.I., M.I.E.E., A.M.I.C.E. and C. J. W. GRIEVESON, M.A., B.Sc. Pp. iv + 9; 6 Figs. Price 6*d.* net.

No. 3. *Light Distribution from Industrial Reflector Fitting No.* 1 (*British Engineering Standard*). By H. BUCKLEY, B.Sc. and C. J. W. GRIEVESON, M.A., B.Sc. Pp. v + 26; 23 Figs. Price 1*s.* net.

No. 4. *Surface Brightness of Diffusing Glassware for Illumination.* By C. J. W. GRIEVESON, B.Sc., M.A. Pp. iv + 9; 3 Figs. Price 9*d.* net.

No. 5. *The Measurement of Mean Spherical Candle-power.* Pp. vi + 40; 20 Figs. Price 1*s.* net.

No. 6. *The Natural Lighting of Picture Galleries.* Pp. viii + 39; 37 Figs. Price 1*s.* 6*d.* net.

No. 7. *Penetration of Daylight and Sunlight into Buildings.* Pp. v + 21; 11 Figs. Price 1*s.* net.

These papers, which are the reports of work carried out under the auspices of the Illumination Research Committee, are primarily intended to be of use to illuminating engineers, but they will be found to contain many points of quite general interest.

No. 1 consists of two sections, terms used in photometry and illumination and terms used in ophthalmology and physiology, the first section having been compiled by Dr J. W. T. Walsh and the second by Dr W. S. Duke Elder.

No. 2 contains the results of measurements of the transmission factor in the case of nine different specimens of window glass, using directed, diffused, and restricted diffused incident light. In an appendix Dr Walsh discusses the reflection factor of a polished glass surface for diffused light.

No. 3 deals with the distribution of light from four reflectors of suitable sizes, using seven different lamps. The type of reflector used is a porcelain-enamelled steel one which "aims at combining the good qualities of the flat cone and of the deep bowl, avoiding the pronounced glare of the one and the patchy illumination of the other."

No. 4 gives an account of the properties of diffusing glassware. Eighteen diffusing bowls of white or pot opal, flashed or cased opal, and translucent alabaster were examined.

No. 5 contains a discussion of the theory of the integrating sphere and hints with regard to its construction and use.

No. 6 deals with the problem of securing adequate and suitable lighting in picture galleries without unpleasant reflections in the glass with which the pictures are protected. A new form of roof lighting, designed for the new Duveen Wing of the National Gallery, Millbank, is described.

No. 7 discusses the various problems connected with the adequacy of light penetrating a room.

ABSTRACTS OF PATENT SPECIFICATIONS

270906. Redfern, C. G. LENSES.

Correction for the astigmatism both of the main lens and the insert of a bi-focal combination is obtained by shaping one of the surfaces to a toric form, before or after fusing the insert in position.

271186. Houghton-Butcher, Manfg. Co., Ltd., and another. LENSES.

Single lenses are so formed as to show a spherical aberration of about 2½ per cent. for rays parallel to the axis incident at about $f/32$. The diaphragm is placed behind the lens at a distance not greater than 15 per cent. of the focal length.

271213. Aldis, A. C. W. DAYLIGHT LAMPS.

The lamp bulb is enclosed in a tapering polygonal reflector, the bottom of which carries a discontinuous colour screen, consisting of two strips of blue-green glass, and two of blue glass, with a central gap.

271611. Tugwood, R. J. REFLECTORS.

A reflecting layer is applied to a glass or marble surface, and coated with celluloid or gelatine, which is then stripped from the base in a vertical position, forming a flexible mirror. A further backing or other protective coating may also be applied.

271970. Jenkinson, E. STEREOSCOPY.

The system comprises two spaced objectives and prisms moved so as to direct alternative images on to adjacent areas upon a film. The prisms are mounted in a box reciprocated by a cam.

271993. Pollard, A. F. C. MICROSCOPES.

The device permits objectives to be changed, and to be held in precise location without employing springs or clamps. A disc carrying six bearing surfaces is secured to the tube of the instrument, at an angle to the axis, and co-operates with six bearing points formed upon the objective carrier. The arrangement is such that, when being removed, the objective cannot damage the slide.

272050. Carpmael, W. SEXTANTS.

The instrument is fitted with an adjustable scale providing for correction for declination of the observed body, dip, solar diameter, parallax, and refraction, and enabling a direct reading of latitude to be obtained.

272371. Thornton, J. E. OPTICAL SYSTEMS.

A moving film is fed around a drum, distortion in the pictures being corrected by a special lens, auxiliary to the projector proper. The lamp with reflector and condenser, all adjustably mounted, may be placed inside the drum, or, as in another arrangement, outside the drum, when the light passes through the film into the drum, and is reflected out axially by a prism.

272799. Hase, R. PYROMETERS; PHOTOMETERS.

An optical pyrometer, which may also be used as a photometer, employs as its comparison field a small area illuminated by a constant lamp, the intensity being varied by the rotation of a disc of graded opacity situated between the source of light and the surface. The comparison surface is preferably in the form of a cross, which when suitably illuminated disappears into the background, while the other remains visible under a graded illumination.

273117. Jessen, A. O. SPEED INDICATORS.

An instrument for use on aircraft includes a fixed telescope, fitted with a prism movable about two axes, by which an object on the ground is kept in sight over a definite angular range of relative movement. The movements are traced upon a chart, from which the linear speed is ascertained, and used to determine wind speed, drift, etc.

273139. Liebl, F. BEVELLING.

The edges of glass articles are bevelled upon a stationary grinding wheel. The work is secured to a rotating holder, which is driven by a tensioned band passing over a pulley similar in shape to the work, which is thus rotated at a varying speed, and moves more rapidly when portions of considerable curvature are against the grinding wheel.

273274. Zeiss, C. LENSES.

Objectives corrected for spherical and chromatic aberrations, astigmatism, and coma consist of three separated components, a single and a cemented collective lens, with a dispersive lens between them. Certain limits expressing the ratios of the various curvatures to each other and to the focal length of the combination are given.

273388. Barr and Stroud, Ltd., and others. SURVEYING.

Images of a pair of stereo-photographs to the same scale are thrown side by side upon a translucent screen by projectors the optical systems of which are so arranged as always to remain in focus. These images are viewed through the screen by a comparator, either of the coincidence or stereoscopic type. The results are transferred to paper to give a scale copy of the features observed.

273682*. Zeiss, C. REFLECTORS.

A silvered reflector for shop lighting, built up by a series of graduated zones, is shaped as a surface of revolution, with its axis coincident with that of the neck of the reflector. The light-emitting aperture is arranged obliquely.

274185. Abraham, R. M. SEXTANTS.

While making the setting by means of one eye applied to the observing telescope, the operator can at the same time see both sides of a graduated scale, the images of which are brought into an eyepiece for observation by the other eye.

274202. Steele, L. J., and another. PROJECTION LAMPS.

Two or more ellipsoidal or similar reflectors, truncated at their rear ends, are placed concentrically so that their foci coincide. The source of light, backed by a hemispherical reflector, is placed at this point. Two or more lamps, each provided with a reflector forming an image at the common focus of the main reflectors, may be employed.

274307. Barr and Stroud, Ltd., and another. RANGEFINDERS.

An accessory is described for use with rangefinders of the self-contained base type, so as to correct the reading of azimuth as for the axis of rotation, that directly obtained having reference to the fixed-angle end of the instrument.

274384. Steele, L. J., and another. PROJECTION LAMPS.

In connection with a projection lamp employing a number of reflectors as in Specification No. 274202, a refracting element is mounted in the path of the unreflected rays. It consists of a double convex lens mounted on a framework of light rods, secured to the reflectors.

R. J. T.

* Specification published before acceptance.

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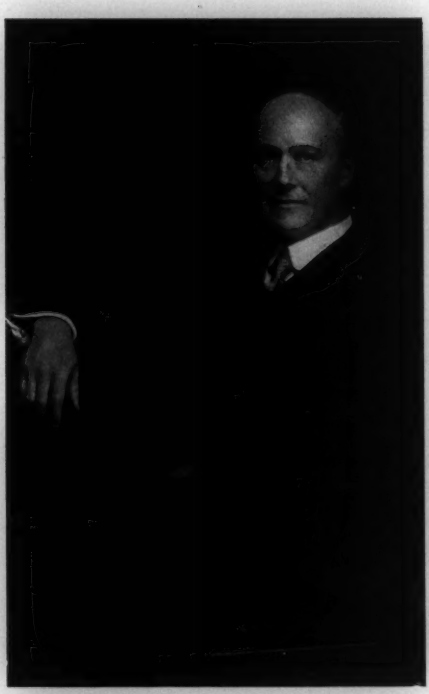

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1927–29